surfaces

- → surface is a continuous two-dimensional object (with measurable area, not volume)
- → it can be described as a record of a curve moving along another curve (and possibly continuosly transforming) in the space
- → parametric equations of a surface:
  - $x=x(u,v)$
  - $y=y(u,v)$
  - $z=z(u,v)$
  - or shortened as $B(t)=(x(u,v),y(u,v),z(u,v))$,
  - where $u,v \in (-\infty,\infty)$ are parameters
- → Usually a surface is limited by parameters
  - $u \in <u_1,u_2>$ and $v \in <v_1,v_2>$,
  - most often $u,v=<0,1>$

surfaces

→ **types of surfaces**

- algebraic – surfaces with known analytic equations
- geometric – surfaces with known geometric definition
- empiric – other surfaces

→ **isocurves of a surface**

- isocurve in *u* direction is a curve achieved by setting *v* parameter to a constant number
- isocurve in *v* direction is a curve achieved by setting *u* parameter to a constant number

→ **intersections of surface with curve**

- none
- point
- more points
- curve (special cases)

→ **intersections of surface with surface**

- none
- point (special cases)
- curve
- more curves
- surface (special cases)

→ **trimming of surfaces**

- each surface can be trimmed by a closed curve which is laying in the surface

→ **tangent plane and normal**

- through each point *A* of a surface there is passing an infinite number of curves → if *A* is a regular point for all the curves, their tangents are in one plane → this plane is a tangent plane of the surface in point *A*
- to find out the tangent plane, it is enough to find out tangents of two curves, e.g. isocurves
- a perpendicular line to the tangent plane is a normal

→ **continuity of a surface**

- each curve passing through point *A* has in this point geometric or parametric continuity of certain degree → the degree of continuity of the surface in this point is the minimum of all this continuities
- to find out the degree it is enough to find out the degree of isocurves

→ **curvature of the surface**

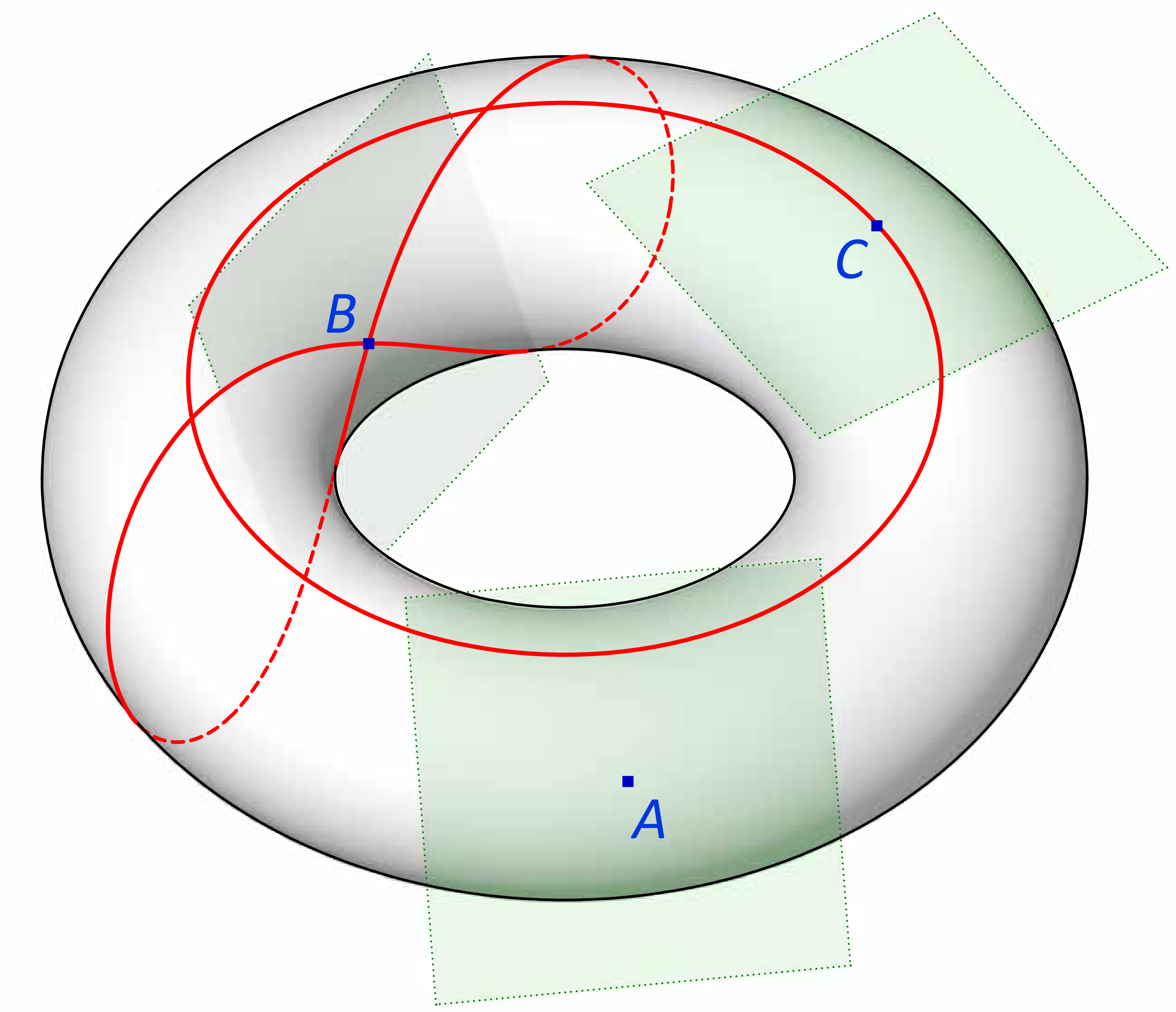
- *normal section* in point *A* is a curve obtained as an intersection of the surface and a plane containing the normal of the surface
- there is and infinite number of normal sections in point *A*
- each of this sections has its curvature in point A (called *normal curvature*)

→ for a surface these curvatures are defined:

- *main curvatures* k_{min} and k_{max} are minimum and maximum normal curvatures; planes of these normal sections are perpendicular
- *mean curvature* $H=\frac{1}{2}(k_{min}+k_{max})$ is arithmetic average of main curvatures
- *Gaussian curvature* $K=k_{min}\cdot k_{max}$ is a product of main curvatures

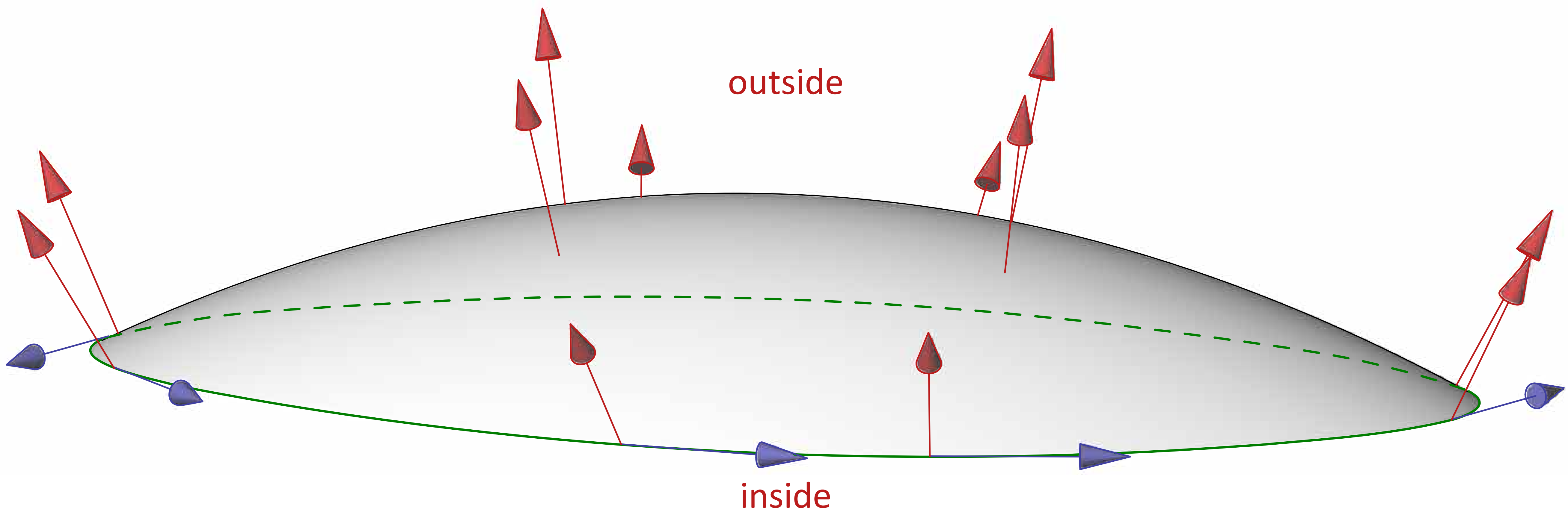
→ **types of points on the surface:**

- elliptic point *A* – tangent plane is not intersecting the surface, main curvatures have the same direction (K>0)
- hyperbolic points *B* – tangent plane is intersecting the surface in a curve with double point *B* (K<0).
- parabolic points *C* – tangent plane is intersecting the surface in a curve with regular point in *C*, one of the main curvature is zero (K=0).



- planar points – tangent plane is toughing in surrounding in a plane, both main curvatures are zero (K=0).

→ (similar to curves' singular points there are multiple points, refraction points, conic points etc.)



→ **orientation of a surface**

- convention: if a surface is observed from outside, curve which is a boundary of the surface is oriented counterclockwise
- orientation is important to model solids (to designed outside and inside of the solid) and rendering (in some programs surfaces are rendered only from outside)

→ straight two dimensional object; is given by:

- three points A, B and C
- or one point P_0 and two direction vectors $\vec{m}$ and $\vec{n}$
- or two intersecting lines

→ plane has mean and Gaussian curvatures equal to zero in all its points

→ parametric equation of plane:

- $B(t)=P_0+\vec{m}u+\vec{n}v$
- where u and $v\in(-\infty,\infty)$ are parameters
- P_0 is the point of surface for u=0 and v=0
- m and $\vec{n}$ are the direction vectors

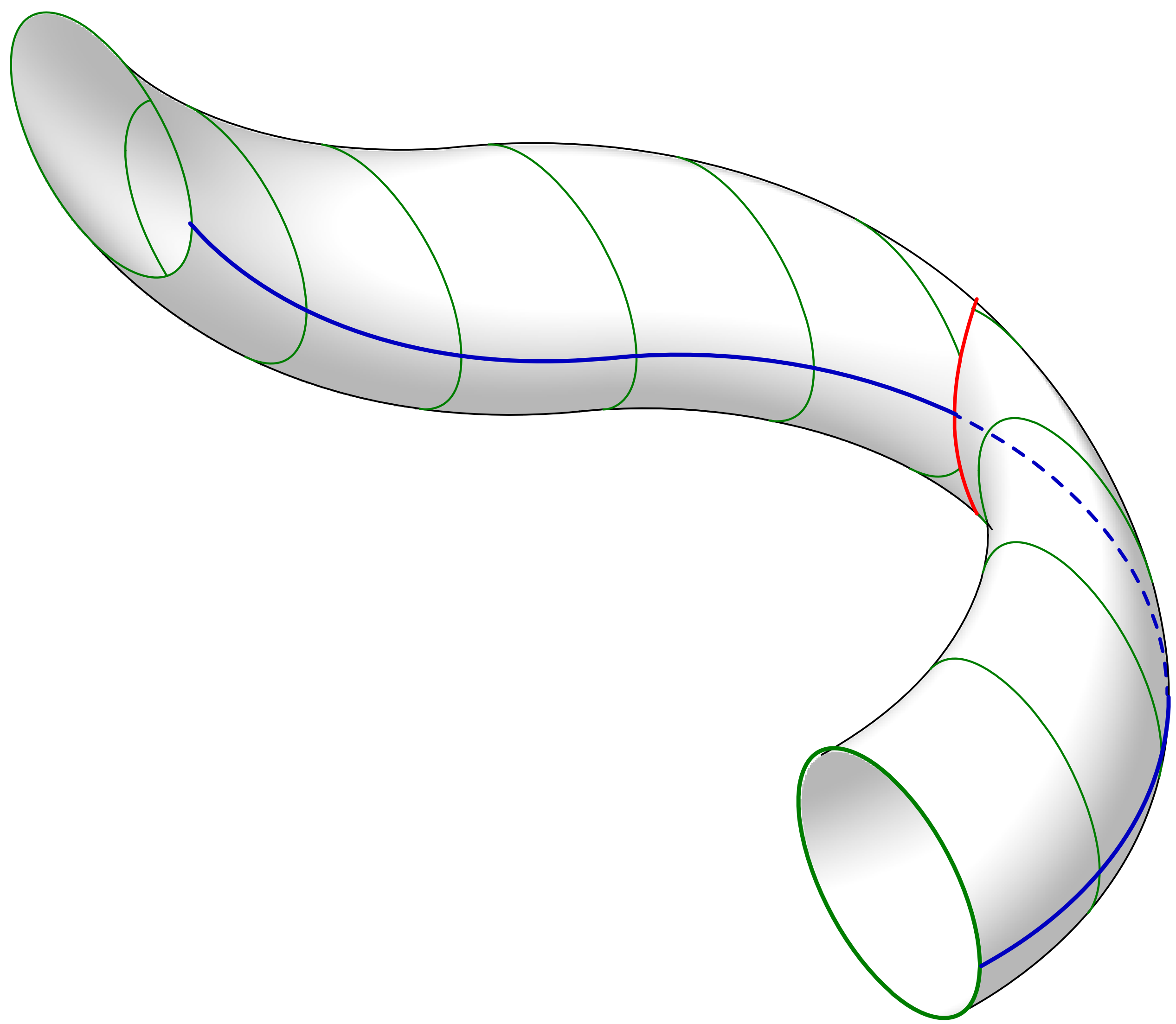
surfaces

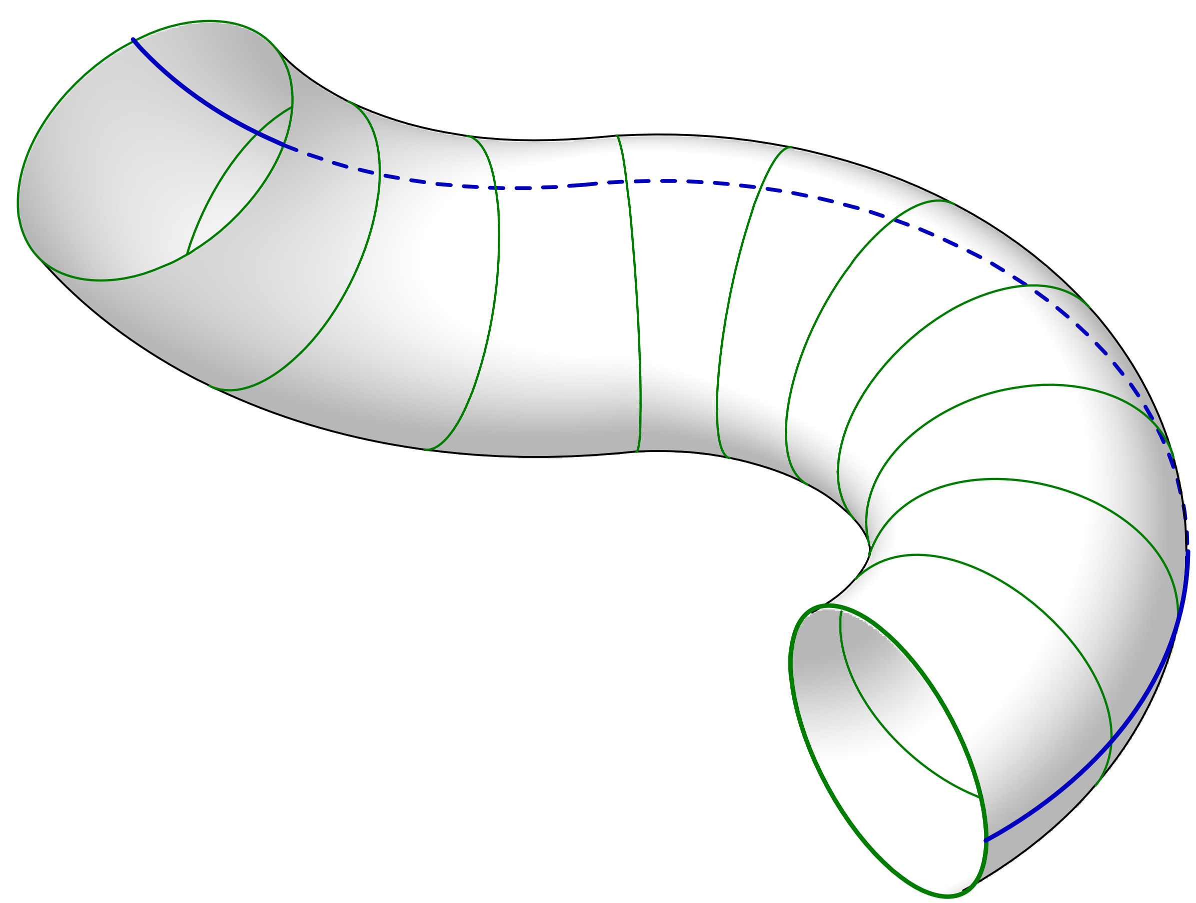
→ CAD systems represent surfaces in the following ways:

- by its geometric definition (translational, swept, rotational etc.) and its geometric properties → e.g. AutoCAD (since 2007), object *`surface`*
- by approximation free form surfaces (NURBS surfaces) → e.g. Rhinoceros, object *`surface`* and *`polysurface`*
- by polygonal mesh representation (most simple, used often as exchange format)

geometrically defined surfaces

→ surface is described as movement of one curve (profile) along another curve(s) (trajectory) with given conditions (parallel movement, rotation etc.); the profile can be continuously transforming



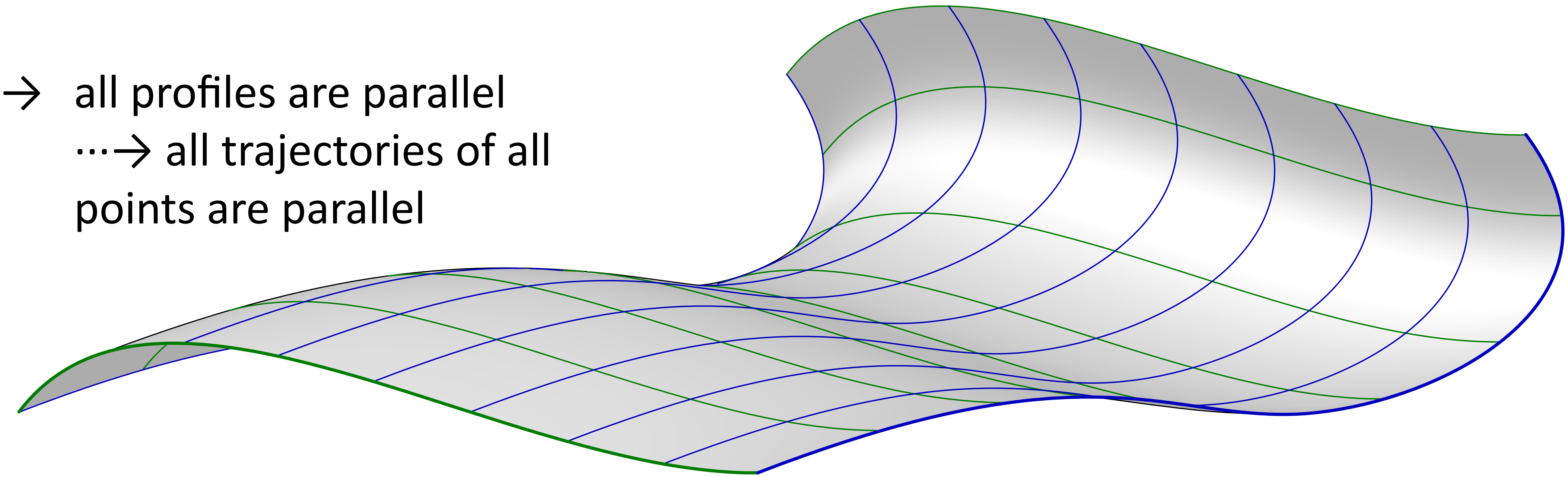


extruded surface ← the same profile (green) and trajectory (blue) → swept surface

geometrically defined surfaces

surfaces	movement conditions: one trajectory	second trajectory	parallel movement	rotation with tangent	rot. around tangent	scaling profile	changing profile shape	special cases
extruded surfaces (translational surfaces)	✓	✗	✓	✗	✗	✗	✗	cylindrical surfaces hyperbolic paraboloid
- with changing profile	✓	✗	✓	✗	✗	✓	○	conical surfaces cuneiform surfaces
swept surfaces	✓	✗	✗	✓	✗	✗	✗	surfaces of revolution, helix and pipe surfaces
- with changing profile	✓	✗	✗	✓	✗	✓	○	channel surfaces
- with two trajectories	✓	✓	✗	✓	✗	✓	○	ruled surfaces, surfaces of pseudorevolution
twisted surfaces	✓	✗	✗	✓	✓	○	○	helix surfaces, möbius stripe

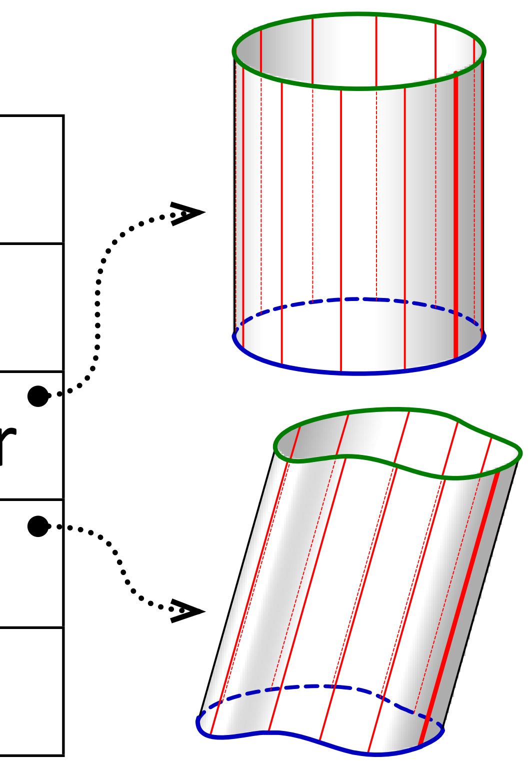
note: ○ shape change and / or non-uniform scaling is possible



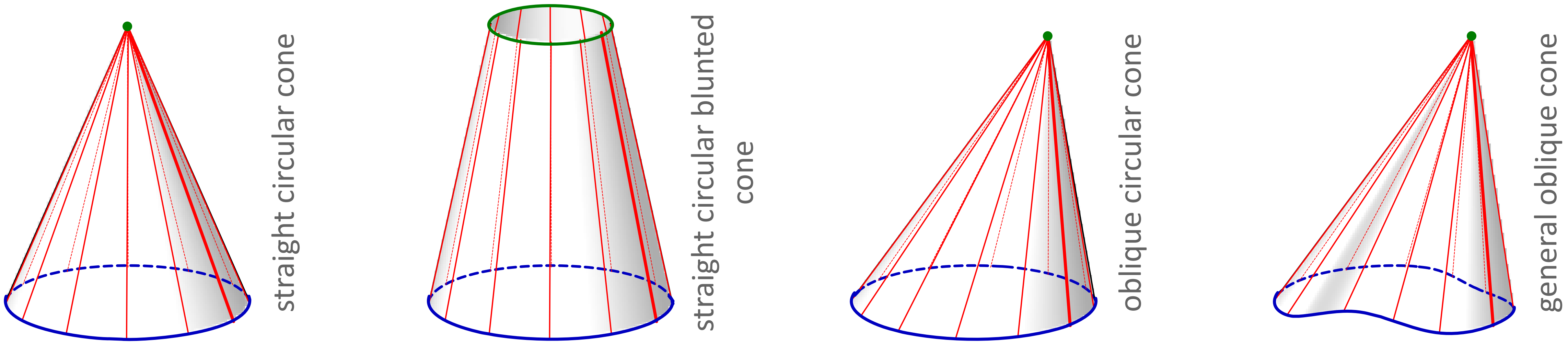
→ all profiles are parallel
···→ all trajectories of all points are parallel

→ basic types of extruded surfaces:

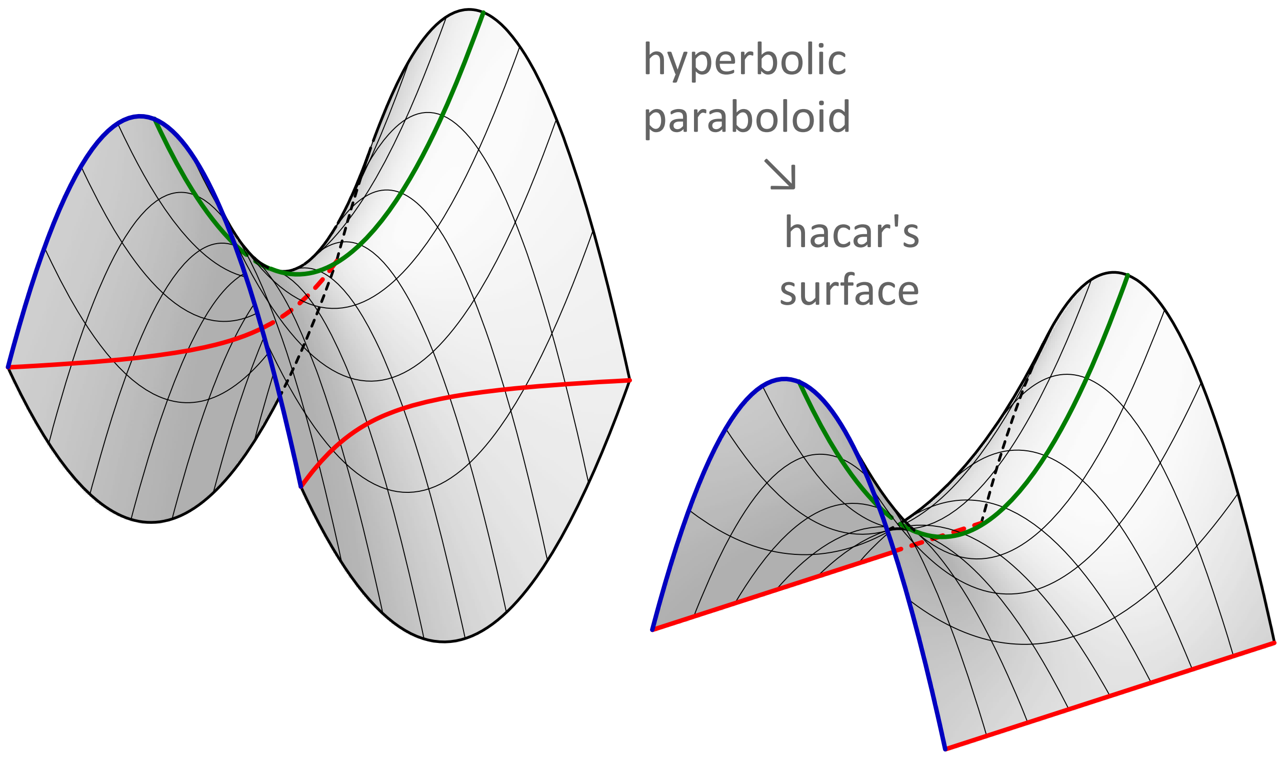
trajectory	**profile**	**resulting surface**
line	line	plane
line	perpendicular circle	straight circular cylinder
line	any curve	cylindrical surface
parabola	parabola	hyperbolic paraboloid

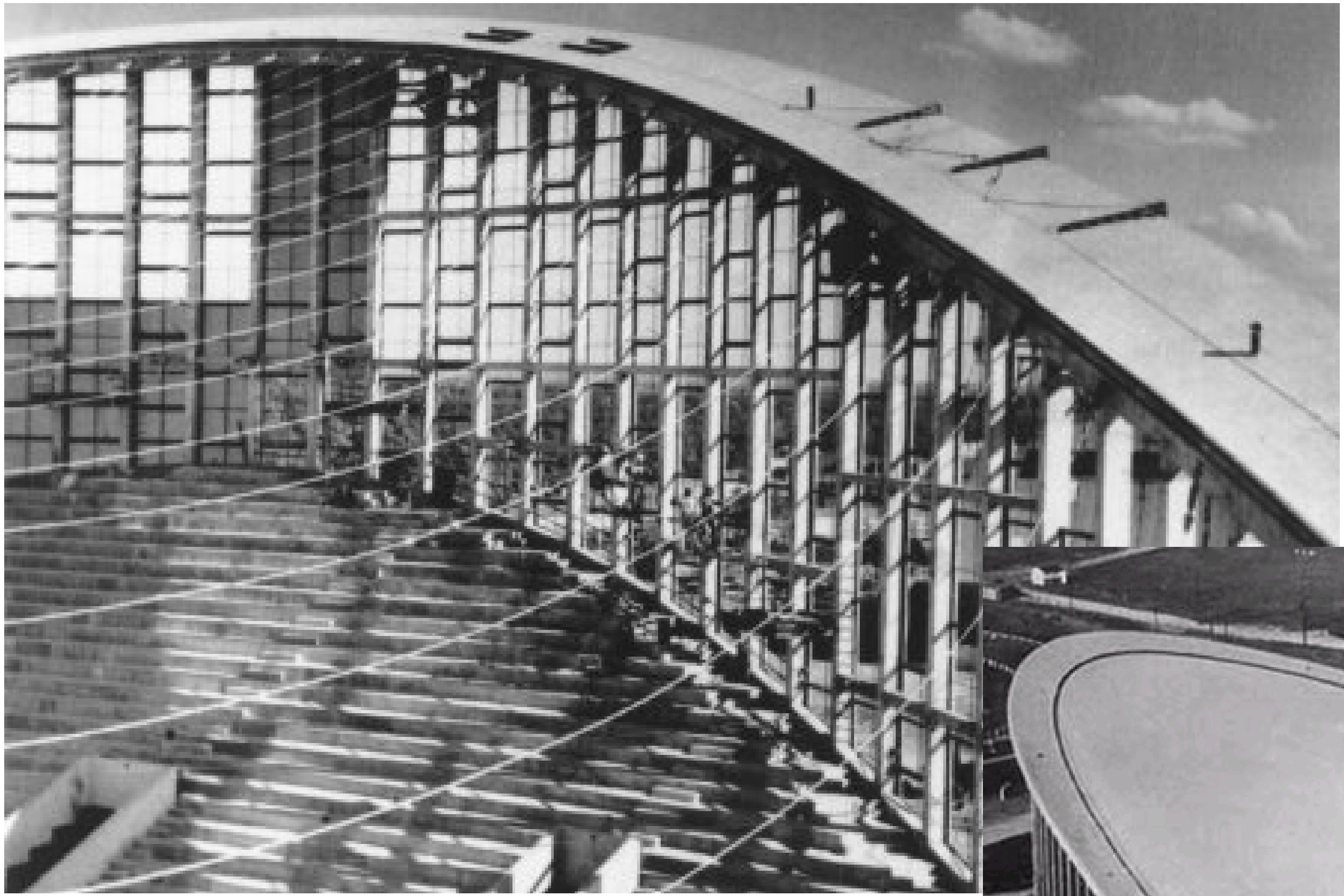


- generalization of extruded surfaces
- examples:
  - conical surfaces



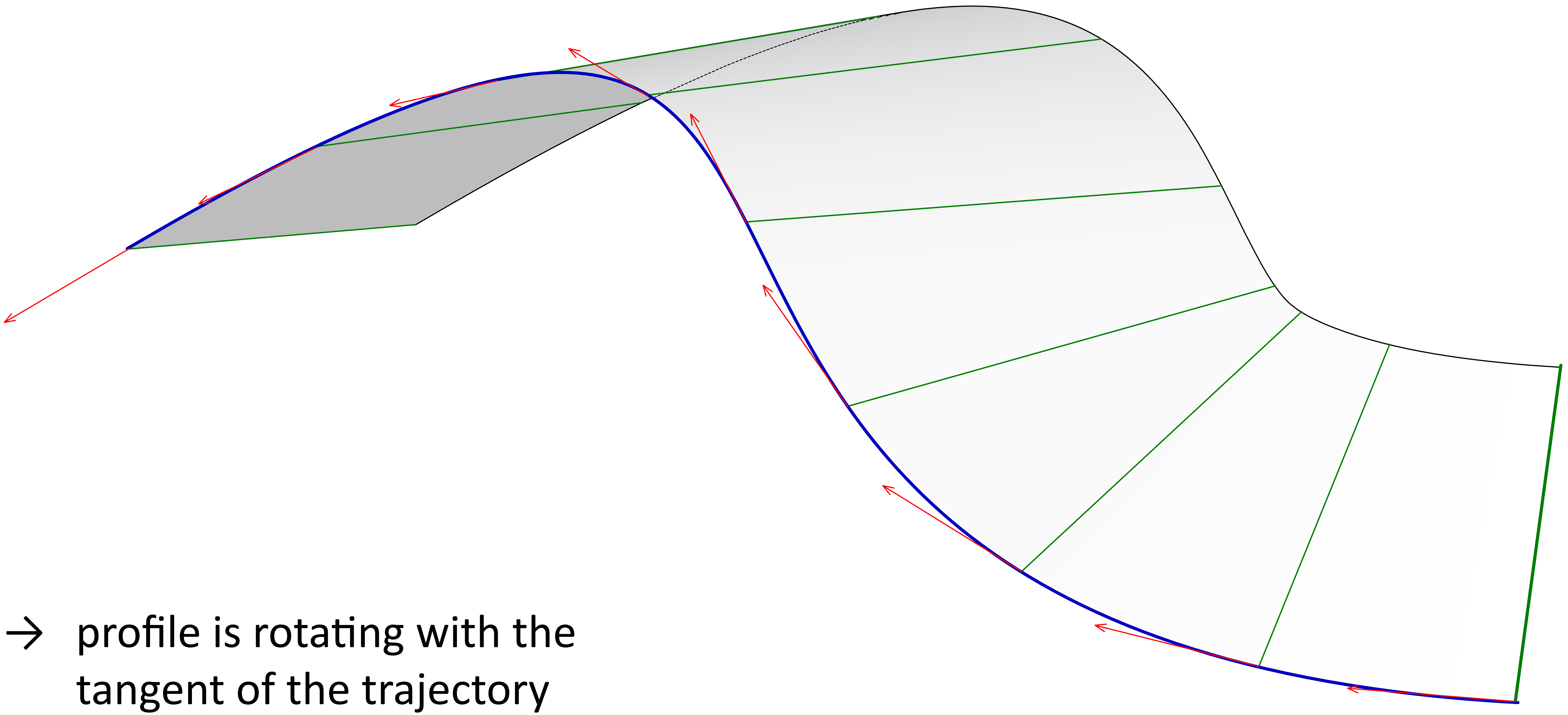
  - cuneiform surfaces





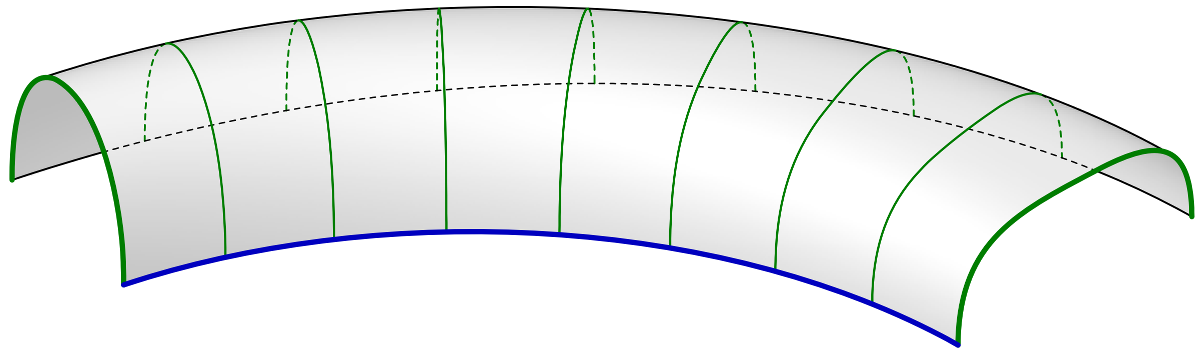
J.S. Dorton Arena (Paraboleum), Raleigh, USA,Maciej Nowicki andWilliam H. Dietrick, 1952 [photo www.arcaro.org and www.c20society.org.uk]

Pavilion V, Brno Trade Fair, Czech Republic, Jaroslav Dokoupil, 2000 [photo ŽS Brno, a. s.]

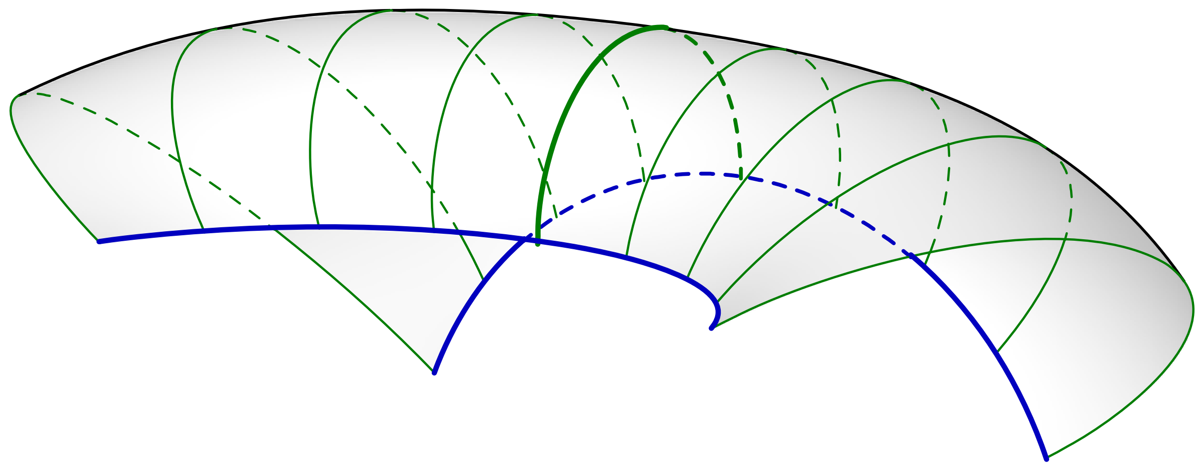


→ profile is rotating with the tangent of the trajectory

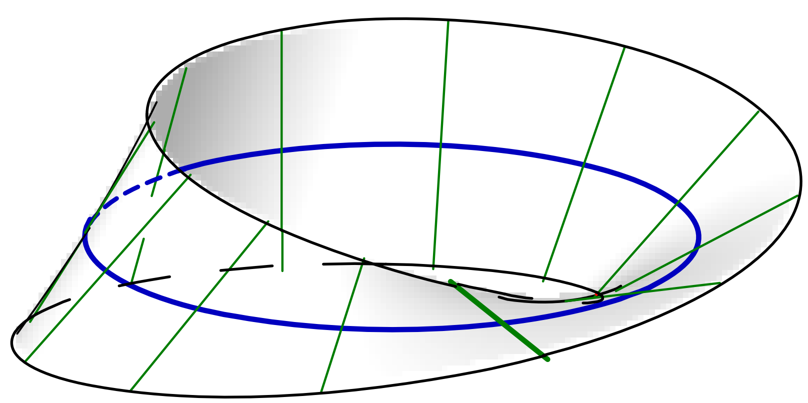
→ swept surfaces with changing profile



→ swept surfaces with two profiles

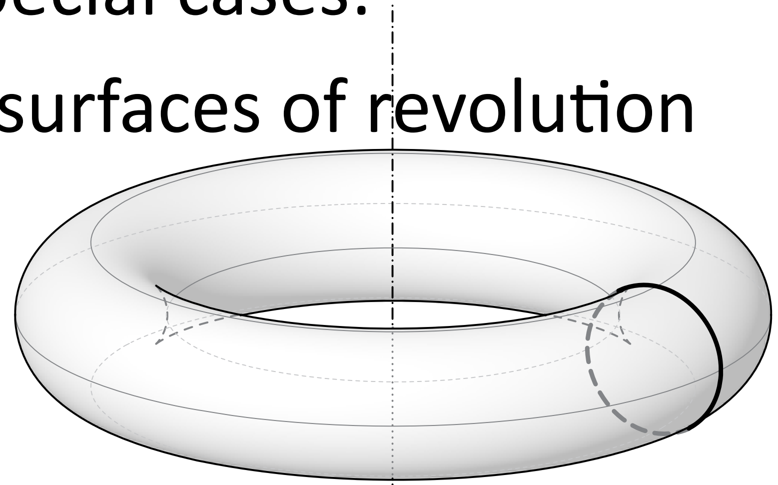


→ twisted surfaces

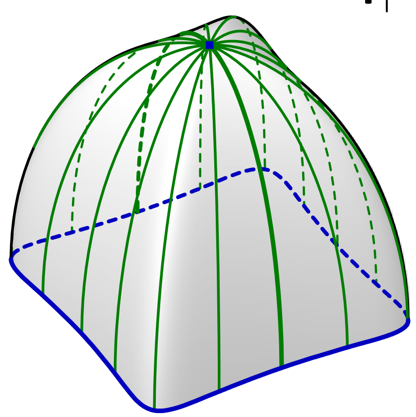


→ special cases:

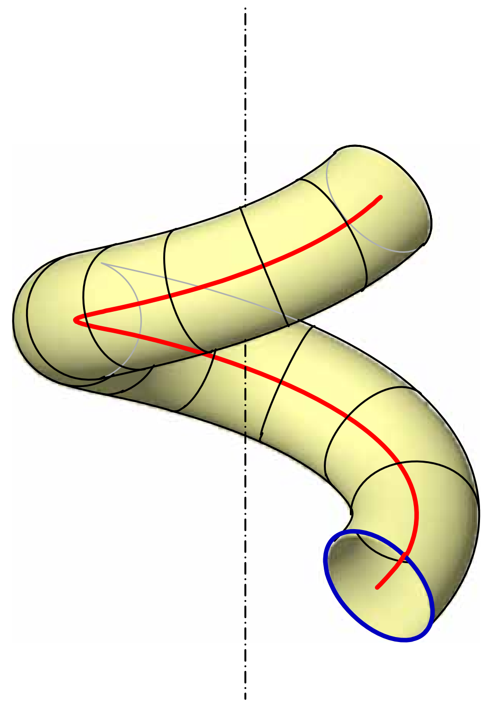
- surfaces of revolution

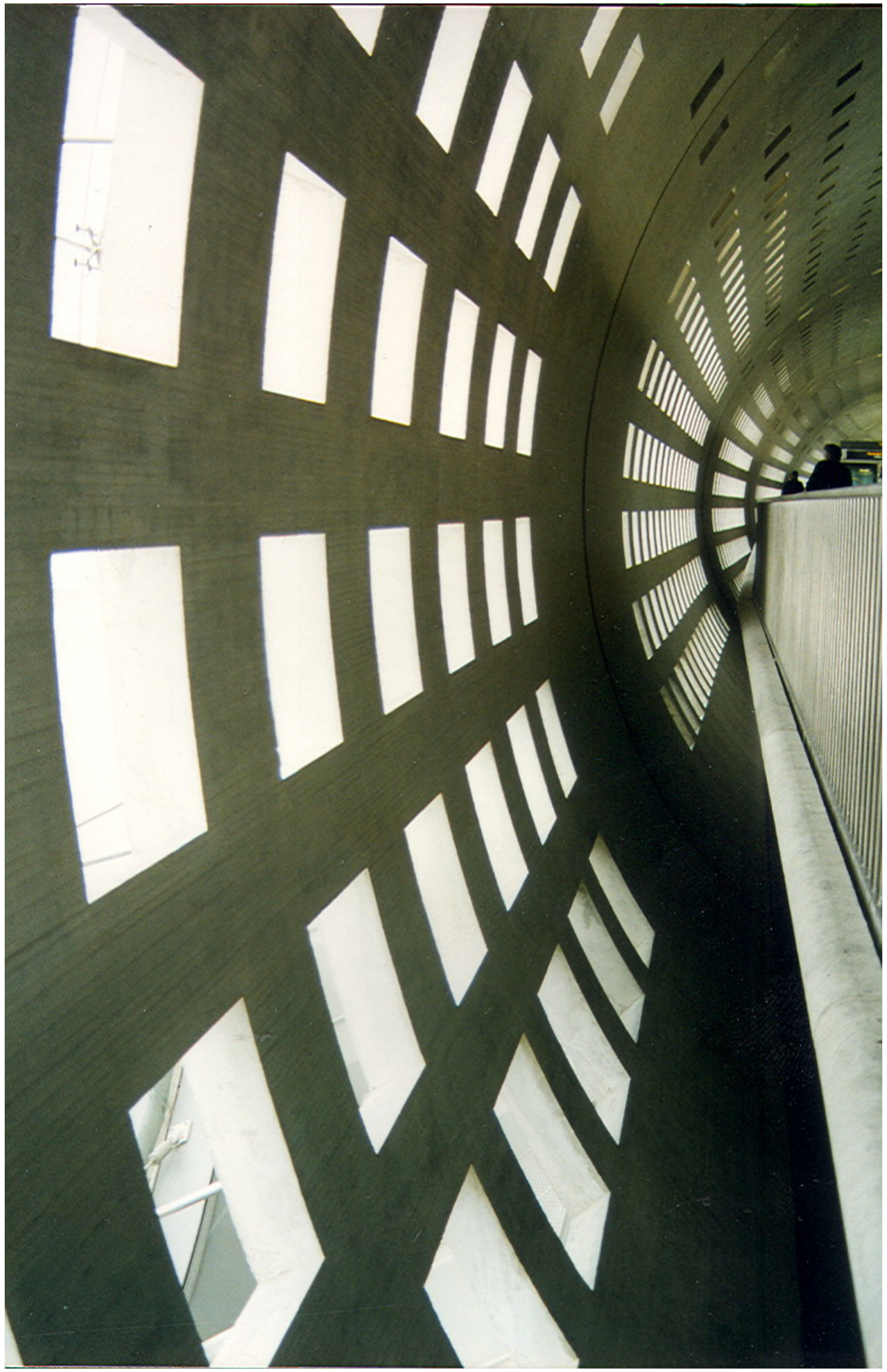


- surfaces of pseudorevolution



- helix surfaces
- ruled surfaces
- Möbius' strips
- pipe surfaces
- channel surfaces





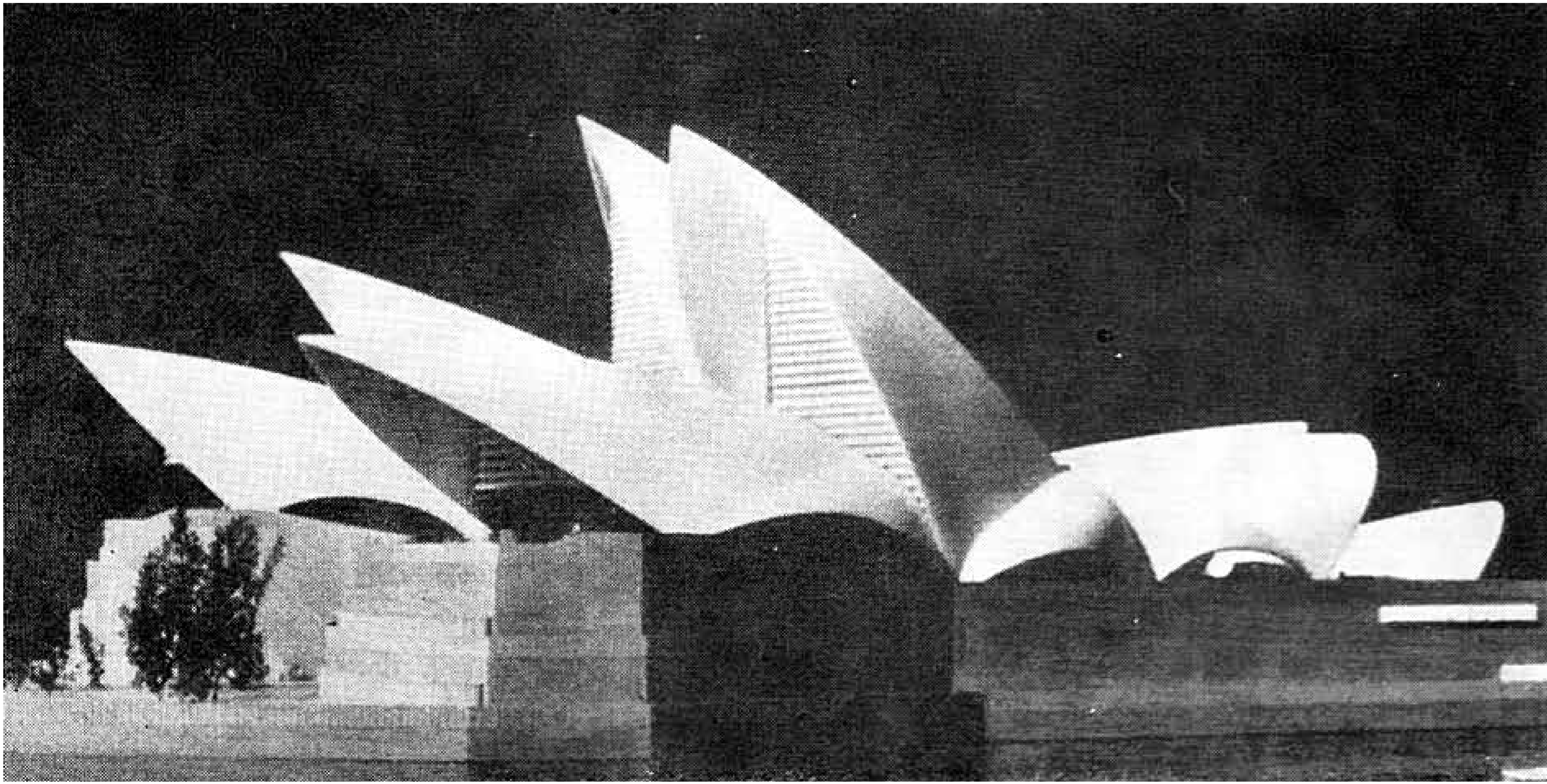
Aéroport Charles de Gaulle
- Aérogare 2F,
Roissy-en-France, France,
Paul Andreu, 1999

[photo by Jan Foretnik, 1999]

ARCAM,
Amsterdam, Netherlands,
René van Zuuk, 2003
[photo by Jan Foretnik, 2005]

Cité des Sciences et de l'Industrie, Parc la Villette, Paris, France, Adrien Fainsilber, 1986

[photo by Jan Foretnik, 1999]



Sydney Opera House, Sydney, Australia, Jørn Utzon, 1973

[photo from Felix Haas: *Architektura 20. století*]

Auditorio de Tenerife, Santa Cruz de Tenerife,Canaria, Spain, Santiago Calatrava, 2003

[photo by R. Liebau, 2006, commons.wikimedia.org]